NAG Toolbox for MATLAB

e02ae

1 Purpose

e02ae evaluates a polynomial from its Chebyshev-series representation.

2 Syntax

```
[p, ifail] = e02ae(a, xcap, 'nplus1', nplus1)
```

3 Description

e02ae evaluates the polynomial

$$\tfrac{1}{2}a_1T_0(\bar{x}) + a_2T_1(\bar{x}) + a_3T_2(\bar{x}) + \cdots + a_{n+1}T_n(\bar{x})$$

for any value of $\bar{x}$ satisfying $-1 \leq \bar{x} \leq 1$. Here $T_j(\bar{x})$ denotes the Chebyshev polynomial of the first kind of degree j with argument $\bar{x}$. The value of n is prescribed by you.

In practice, the variable $\bar{x}$ will usually have been obtained from an original variable x, where $x_{\mathrm{min}} \leq x \leq x_{\mathrm{max}}$ and

$$\bar{x} = \frac{((x - x_{\mathrm{min}}) - (x_{\mathrm{max}} - x))}{(x_{\mathrm{max}} - x_{\mathrm{min}})}$$

Note that this form of the transformation should be used computationally rather than the mathematical equivalent

$$\bar{x} = \frac{(2x - x_{\mathrm{min}} - x_{\mathrm{max}})}{(x_{\mathrm{max}} - x_{\mathrm{min}})}$$

since the former guarantees that the computed value of $\bar{x}$ differs from its true value by at most 4ϵ, where ϵ is the ***machine precision***, whereas the latter has no such guarantee.

The method employed is based on the three-term recurrence relation due to Clenshaw 1955, with modifications to give greater numerical stability due to Reinsch and Gentleman (see Gentleman 1969).

For further details of the algorithm and its use see Cox 1974 and Cox and Hayes 1973.

4 References

Clenshaw C W 1955 A note on the summation of Chebyshev series *Math. Tables Aids Comput.* **9** 118–120

Cox M G 1974 A data-fitting package for the non-specialist user *Software for Numerical Mathematics* (ed D J Evans) Academic Press

Cox M G and Hayes J G 1973 Curve fitting: a guide and suite of algorithms for the non-specialist user *NPL Report NAC26* National Physical Laboratory

Gentleman W M 1969 An error analysis of Goertzel's (Watt's) method for computing Fourier coefficients *Comput. J.* **12** 160–165

5 Parameters

5.1 Compulsory Input Parameters

1: **a**(**nplus1**) – **double array**

a(i) must be set to the value of the ith coefficient in the series, for $i = 1, 2, \ldots, n + 1$.

2: **xcap – double scalar**

$\bar{x}$, the argument at which the polynomial is to be evaluated. It should lie in the range -1 to $+1$, but a value just outside this range is permitted (see Section 6) to allow for possible rounding errors committed in the transformation from x to $\bar{x}$ discussed in Section 3. Provided the recommended form of the transformation is used, a successful exit is thus assured whenever the value of x lies in the range $x_{\min}$ to $x_{\max}$.

5.2 Optional Input Parameters

1: **nplus1 – int32 scalar**

Default: The dimension of the array **a**.

the number $n+1$ of terms in the series (i.e., one greater than the degree of the polynomial).

Constraint: **nplus1** ≥ 1.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **p – double scalar**

The value of the polynomial.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

ABS(**xcap**) $> 1.0 + 4\epsilon$, where ϵ is the ***machine precision***. In this case the value of **p** is set arbitrarily to zero.

ifail $= 2$

On entry, **nplus1** < 1.

7 Accuracy

The rounding errors committed are such that the computed value of the polynomial is exact for a slightly perturbed set of coefficients $a_i + \delta a_i$. The ratio of the sum of the absolute values of the δa_i to the sum of the absolute values of the a_i is less than a small multiple of $(n+1)$ times ***machine precision***.

8 Further Comments

The time taken is approximately proportional to $n+1$.

It is expected that a common use of e02ae will be the evaluation of the polynomial approximations produced by e02ad and e02af.

9 Example

```
a = [2;
     0.5;
     0.25;
     0.125;
     0.0625];
xcap = -1;
[p, ifail] = e02ae(a, xcap)
```

```
p =
    0.6875
ifail =
           0
```